

Kähler versus Non-Kähler Compactifications

Melanie Becker¹, Keshav Dasgupta²

¹ *Department of Physics, University of Maryland, College Park, MD 20742-4111*

² *Department of Physics, Varian Lab., Stanford University, Stanford CA 94305-4060*

melanieb@physics.umd.edu, keshav@itp.stanford.edu

Abstract

We review* our present understanding of heterotic compactifications on non-Kähler complex manifolds with torsion. Most of these manifolds can be obtained by duality chasing a consistent F-theory compactification in the presence of fluxes. We show that the duality map generically leads to non-Kähler spaces on the heterotic side, although under some special conditions we recover Kähler compactifications. The dynamics of the heterotic theory is governed by a new superpotential and minimizing this superpotential reproduces all the torsional constraints. This superpotential also fixes most of the moduli, including the radial modulus. We discuss some new connections between Kähler and non-Kähler compactifications, including some phenomenological aspects of the latter compactifications.

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The Calabi-Yau (CY) compactifications of Candelas et al. [1] have led to some major progress in our understanding of string theory vacua. Compactifying the heterotic string on such manifolds results in four-dimensional models with minimal supersymmetry (susy). In terms of the corresponding two dimensional non-linear sigma model, we demand conformal invariance so that all the tadpoles vanish and the string equations of motion are satisfied. In this way we recover again CY spaces. For the bosonic case this will give us the model studied in [2]. By definition CY manifolds are Kähler and have a vanishing first Chern class. By Yau's theorem therefore, for a *given* complex structure and a *given* cohomology class of the Kähler form there is a *unique* Ricci-flat metric with $SU(3)$ holonomy¹.

Generically, when considering ordinary CY compactifications of the heterotic string theory the three-form background fluxes (at weak coupling and constant dilaton) are equal to zero. The four-dimensional spacetime is Minkowski and therefore has zero cosmological constant. Although susy would also allow anti de-Sitter solutions, here only the Minkowski solution is realized. The cancellation of the two-loop sigma model beta function puts a strong constraint on the vector bundle, namely it has to be identified with the tangent bundle, implying that the three form is in the cohomology classes of the manifold². Further, by the Uhlenbeck-Yau theorem, there is an essentially unique choice of vector field for any *given* holomorphic stable vector bundle satisfying the Donaldson-Uhlenbeck-Yau (DUY) equations.

This attractive scenario however is clouded by some inherent problems which are related to the degeneracy of string vacua. Essentially there are two different degeneracies appearing in string theory compactifications. First, there are thousands of CY manifolds that could be potential solutions to the low energy effective theory. Second, once we choose a *particular* CY manifold, there are many different moduli associated with the complex structure and the Kähler structure deformations of the manifold. All these moduli are unstable at tree level and thus lead to a situation that is unattractive for phenomenology. It turns out that the radial modulus of the CY is one of the Kähler moduli. Therefore, when this field is not stabilized the CY will runaway to infinite size. This ruins the whole consistency of the compactification scenario [4].

¹ Ricci flatness is not an essential property of the compactifying manifold as has been demonstrated in [3]. We can restore Kählerity without having a Ricci flat metric (a field redefinition relates them). For the non-Kähler manifolds, however, we can never have a Ricci flat metric.

² This is a sufficient condition, but not a necessary one. We shall discuss this in more detail as we go along.

One way to remove the degeneracy for a given CY is to relax the restriction on the fundamental two-form J by allowing manifolds that have $dJ \neq 0$, i.e spaces that are non-Kähler. These more general compactifications were first discussed in detail in [5],[6] and [7]. At first sight it may not be apparent at all how one could remove degeneracies by going to non-Kähler manifolds. This will be explained below. But first observe that breaking the Kähler condition is not straightforward, as one can show that in the absence of background fluxes *and* warped metrics, the generic solution is always Kähler. Therefore for a non-Kähler manifold to be a solution of the equations of motion one has to switch on non trivial three form fluxes \mathcal{H} , which will essentially play the role of a torsion. The torsion is not closed because an embedding of the spin-connection into the gauge-connection is not allowed, as this would lead back to CY manifolds. Furthermore, the dilaton is generically non constant and related to the warp factor of the underlying manifold.

The first concrete example of such manifolds was constructed in [8] and [9] by duality chasing a particular model of the general class of M-theory compactifications with non vanishing fluxes considered in [10]. The manifold constructed is a T^2 bundle over a four dimensional $K3$ base. In [9] and [11] many properties of this manifold were explicitly found by going to the orbifold limit of the $K3$ base. So for example, the manifold is compact, complex, has a vanishing first Chern class and $SU(3)$ holonomy. It was observed in [12], [11] that the Betti numbers of this manifold are different from the Betti numbers of a simple product $K3 \times T^2$ (appearing for vanishing fluxes), implying a topology change. The topology change is achieved by considering an additional *twist* (along with the flux) so that a consistent solution that preserves minimal susy in four dimensions is obtained. Due to this fact it is *not* possible to construct this manifold directly in the heterotic theory using a supergravity analysis. This has been explained in [13]. However the duality chasing that we performed miraculously takes the topology change into account, so that a consistent non-Kähler manifold appears on the heterotic side. See [9] and [13] for a more detailed analysis of this. In fact, the *twist* that we expect on the heterotic side is actually one component of the G -flux in M-theory.

An immediate advantage that compactifications on manifolds with torsion have is moduli stabilization at tree level. The fluxes give rise to a potential that stabilizes *all* the complex structure moduli [9], the radial modulus and some of the remaining Kähler moduli [13], [11]. This stabilization can be understood in terms of a superpotential first constructed in [13] and verified later by dimensional reduction in [14]. Contrary to popular belief, the superpotential is complex and is given by $W = \int (\mathcal{H} + idJ) \wedge \Omega$. Here J is the

fundamental two form which may not necessarily be integrable for an arbitrary choice of fluxes and Ω is the unique holomorphic (3,0) form wrt the almost complex structure that characterizes these manifolds. The form of this superpotential implies that the dynamics of the heterotic theory compactified on these non-Kähler manifolds can be described by a *complex* three form \mathcal{G} . This three form is anomaly free and gauge invariant (see [11] and [13] for a derivation of this) and therefore can be used to construct the scalar potential for all the moduli from its kinetic term $\int |\mathcal{G}|^2$. This potential incorporates terms that are of higher order in α' (see [15] for details). Notice, that the no-scale structure of the potential is broken in a rather interesting way. The anomaly free three form can be shown to depend secretly on the radial modulus (and some of the Kähler moduli) by solving the anomaly condition [13]. The non-trivial radial dependence comes from the fact that the Bianchi identity incorporates the three form on both sides of the equation and therefore can be solved iteratively order by order in α' [11], [13]. In [13] a simple analysis was performed to evaluate the radius of the non-Kähler manifold. It was shown that the $K3$ base can be made large enough by choosing large flux densities, but the fiber generically has a size of order α' . This again implies that a simple supergravity analysis cannot be performed directly in the heterotic theory and duality chasing needs to be performed ³.

There are a couple of questions one might ask at this point. One immediate one would be whether we can choose *any* fourfold in M-theory, or whether this choice is restricted. As in the usual relation between compactifications of F-theory on a fourfold and the heterotic string on a threefold there is, of course, the restriction that the fourfold should be an elliptic fibration. Observe that the particular fourfold that was chosen for duality chasing i.e $K3 \times K3$, has an orientifold description on the type IIB side, which makes the description easier. The orientifold operation actually involves three actions: world sheet parity (the usual orientifold), fermion number reversal and space reversal (the orbifold action). The presence of these three actions guarantees that a set of U-dualities will take us to the heterotic string [8],[9] and not bring us back to the strongly coupled type IIB theory. This aspect was used in [15] to construct new examples of non-Kähler manifolds that have non

³ Although the duality chasing works for most cases, sometimes it may not give the complete answer. For example, the full heterotic Bianchi identity, or the scalar potential do not follow from simple duality chasing. The derivation of these require higher order α' corrections of the action and the T-duality rules, that do not appear in the supergravity approximation. These subtleties have been explained in [11] and [15].

zero Euler characteristics. The original non-Kähler manifolds constructed in [8],[9] have zero Euler characteristics [12], [11].

The next question is the choice of fluxes. Anomaly cancellation in M-theory implies that fluxes are necessary, if compactifications on manifolds with non-vanishing Euler characteristics are being considered [10], [16]. An alternative picture in which fluxes are traded with space filling branes and their consequent effect on the geometry is discussed in [15]. From this discussion one could naively conclude that one should *always* get a torsional compactification on the heterotic side and the simpler CY compactifications are ruled out. Interestingly, it turns out that for some special choice of fluxes, we will get back an ordinary CY compactification. Let us elaborate this a little bit. More details will appear elsewhere. There are two different types of fluxes that we can choose on the M-theory side: one that is positioned over the full fourfold and the other that is *localized* at the fixed points of the fourfold. These fixed points are the points where the T^2 fibration degenerates. As discussed in detail in [8], [9], the non-localized fluxes appear on the type IIB side as the NS and RR three form fields. If these fluxes are not vanishing we eventually obtain the non-Kähler manifolds with torsion three form on the heterotic side after further U-dualities. The localized fluxes, on the other hand, become the seven brane gauge fields on the type IIB side [9], [11], which under further U-dualities become the heterotic gauge fields. These gauge fields originate in M-theory by decomposing the G-fluxes in terms of the localized (normalizable) harmonic (1,1) forms near each singularity [17]. In the presence of non-localized G-fluxes the localized (1,1) forms themselves change by the backreaction of the fluxes on the geometry. This can be worked out with some effort [17], but will be ignored in the following. Generically in the presence of both fluxes we would get non-Kähler spaces with torsion. Furthermore, the background G-fluxes *warp* the geometry in some special way [8],[9],[11]. The equation for the warp factor was given in [8],[9]. What happens if we choose only the localized fluxes? It is easy to see that in type IIB theory we will get gauge fields on the seven branes and no three form background fluxes. The anomaly cancellation condition will put some restriction on the total instanton numbers of these gauge fields. This is of course the usual restriction that we expect also on the heterotic side. Therefore we seem to recover ordinary CY compactifications, except for the fact that in the presence of fluxes in M-theory we will typically get a warped metric. This would naively ruin the $dJ = 0$ property. However a careful analysis reveals that the warp factor equation e.g. on the type IIB side, is in fact proportional to $\Omega_3(\omega) - \Omega_3(A)$, where Ω_3 is the Chern-Simons three form (see [8] and [9] for the derivation of this). Therefore if

we *embed* the spin connection ω into the gauge connection A we will recover a trivial warp factor and the manifold will become Kähler! This is precisely the reason behind embedding the spin-connection into the gauge connection. Here we have rederived this property from M-theory by demanding the consistency of CY compactifications.

There is another rather interesting aspect that comes to mind at this point. Imagine that we do not turn on the non-localized gauge fluxes and at the same time do not allow the standard embedding. Then the naive expectation would be that we should get a non-Kähler manifold with the non-Kählerity coming precisely from the difference $\Omega_3(\omega) - \Omega_3(A)$. This would seem to contradict the result of [18] where it was found that a fractional gauge Chern-Simons term can appear in an ordinary CY compactification. However this apparent puzzle can be resolved by taking the background *gaugino condensate* into account⁴. To see how this helps we need to back up a little for more generality.

The complex three form that appears in the heterotic theory in the presence of torsion has to be imaginary self-dual (ISD) to preserve supersymmetry in four dimensions. This implies the background equation $dJ = *\mathcal{H}$, where $*$ is the Hodge duality in six dimensions (over the non-Kähler manifold). This equation is more general than the constraint derived in [5],[6], [7] and it reduces to the known form when the manifold is complex (see [13] for a derivation of this fact). This equation makes the non-Kähler nature of the manifold manifest. Observe that if we scale the metric then the three form \mathcal{H} scales linearly too. On the other hand from the Bianchi identity we observe, that the three form does not scale (at least to the lowest order in α'). This implies that the radial modulus should get stabilized, as we saw earlier. This argument, although correct, is rather naive at this point. The fact that the Bianchi identity does not scale is only true for the Kähler case. In the non-Kähler case since the three form appears on both sides of the identity, the issue is more subtle. Therefore the correct way to study the potential for the radial modulus would be to evaluate the three form flux order by order in α' and use the kinetic term to calculate the potential. This was done in [13], [15].

Coming back to the torsional constraint, we see that in the absence of three form fluxes the manifold can still become non-Kähler via the relation $dJ = *\alpha'[\Omega_3(\omega) - \Omega_3(A)]$. In [18] the $\Omega_3(\omega)$ term was cancelled by one of the Chern-Simons terms of the gauge fields. It turns out that one can make $dJ = 0$ using the gaugino condensate contribution to the

⁴ The gaugino condensate contributes to the (3,0) and the (0,3) part of the threeform, and therefore breaks susy.

superpotential. The gaugino condensate will change the torsional equation by an additional term (for a derivation of this see for example [19]). This additional term can be used to cancel the remaining gauge Chern-Simons part and we recover the Kähler property⁵.

Therefore we see that Kähler compactifications are a special case of the more general non-Kähler compactifications with torsion (at least those non-Kähler compactifications that could be constructed from F-theory using duality chasing). To summarize: the most generic superpotential governing these backgrounds is complex and only for the special case $dJ = 0$ we recover the superpotential proposed in [21].

Our next goal is to understand the fibration structure of our non-Kähler manifold. The precise metric of the fiber torus has been worked out in [9], [11]. As we discussed earlier, there is a change in Betti numbers when we go from $K3 \times T^2$ to the non-Kähler space. It turns out that this change of “topology” can be understood from a *brane-box* configuration in the type IIB theory. The brane-box divides the region in two parts: one that is inside the box and the other outside. The walls of the box are made out of NS5-branes which are actually sources of NS flux. The RR three form flux can be obtained from the T-dual twist of a type IIA configuration (see [15] for a detailed analysis of this scenario)⁶. Under a set of U-dualities each side of the brane box transforms into a Taub-NUT space which reproduces the fibration structure. The fact that the metric of the system works out correctly has been checked in [15]. To see how the superpotential works out from the brane box configuration we observe that the RR three form sources can also be replaced by D5 branes (forming, say, the two other sides of the box). These D5 branes become, under a set of U-dualities, NS5 branes wrapping some two cycles of the non-Kähler space whose fibration structure is determined from the U-dual brane box. The NS5 branes show a jump of the \mathcal{H} charge precisely as $\mathcal{H} = - * dJ$ [22], and therefore contribute the complex dJ part of the superpotential. More details on this will appear in

⁵ A more “dynamical” way to achieve this would be to take our proposed superpotential and solve for dJ using the various contributions (tree level, perturbative and non-perturbative). If $dJ = 0$ with integrable J we get Kähler CY compactifications. All other cases, i.e when $dJ \neq 0$ with J integrable or non-integrable, will correspond to non-Kähler compactifications. As we saw earlier, for this case the radius is stabilized at tree level. For the Kähler case, the radius is fixed non-perturbatively [20],[18]. In both cases the σ -model conformal invariance is restored at *this* particular radius.

⁶ As an additional advantage we get non constant three form fluxes, as opposed to the constant fluxes of [8],[9].

a future publication. Thus we can reproduce the full structure of the non-Kähler space using a brane configuration. This should come as no surprise because far away from the brane box configuration there is no distinction from the geometrical picture and the brane picture. There are some subtleties in this construction primarily related to the hidden orientifold nature of the system. The box configuration survives this projection. But one also has to take into account the F-theory monodromies to reproduce the complete gauge bundles. These monodromies appear as a stringy cosmic string in our model (again for more details the readers are advised to look in [15]).

Before moving ahead, we still have to clarify the reason of why these manifolds have an $SU(3)$ holonomy (wrt the torsional connection). The existence of minimal susy in four dimensions gives us a necessary condition that is, however, not sufficient. A slightly more stronger argument⁷ will be to observe that if we choose the metric so that the cube of J is the product of the holomorphic three form Ω and its dual, then the holomorphic connection that respects the metric will also respect the holomorphic three form and therefore will have $SU(3)$ holonomy. The profound aspect of Yau's theorem is that, in the CY case, the metric can be chosen to be Kähler in addition to the above property. Since there is no Kähler metric for the more general non-Kähler compactifications, we have nothing comparable to prove. We do, however, need another principle for choosing the particular metric we have chosen out of an infinite dimensional space of metrics. The above criterium is satisfied by our choice of metric.

We can also explain this using the torsion classes \mathcal{W}_i (with $i = 1, \dots, 5$) of [23] and [24]. In this classification, a non-Kähler manifold that preserves an $SU(3)$ holonomy will have to necessarily satisfy the equation $2\mathcal{W}_4 + \mathcal{W}_5 = 0$, with an additional condition $\mathcal{W}_1 = \mathcal{W}_2 = 0$, so that the complex structure is integrable. For more details on the physics aspects of this see for example [23]. For our non-Kähler manifold the torsion classes have been worked out in [11] and for the Iwasawa manifold, in [23]. Using this analysis, it can also be shown that one *cannot* define any Kähler metric on these manifolds (see [12],[11] for more details). Therefore these manifolds are explicitly non-Kähler.

So far our discussion has been mostly restricted to non-Kähler manifolds that have zero Euler characteristics. Having zero Euler characteristics is not much of a problem, of course, because we are not embedding the spin connection into the gauge connection. The reason why we would like to look for more general manifolds with non-zero Euler characteristics

⁷ We thank Paul Green for providing the following argument.

is purely to extend our understanding of these manifolds. This may serve as a new and interesting direction in mathematics and may also turn out to be phenomenologically more attractive.

The first interesting example of such a manifold is four dimensional and can be described in terms of a $K3$ manifold with torsion. The metric has been worked out in [5], where it was shown to pick up an overall conformal factor from the back reaction of the fluxes on the geometry. One may wonder if it is possible to obtain this manifold from duality chasing in F-theory. Indeed this has been achieved in [15] (at least for the non-compact case). Therein it was shown that the metric is related to a Gimon-Polchinski kind of model with the axion-dilaton τ fixed at a *particular* value. In this sense this is different from our earlier examples where we had a vanishing axion-dilaton and therefore F-theory was at *constant* coupling [25]. Now since the axion-dilaton is non-trivial, there would be sizable non-perturbative corrections to the model. These corrections actually convert the intersecting orientifold planes and branes to smooth hyperbolas. For details on this see [26] for the case of zero torsion and [15] for the case with torsion. An alternative way to appreciate this would be to observe that the metric of a torsional $K3$ is precisely the metric of a NS5 brane at a point on the $K3$. Therefore under a set of U-dualities the system maps to a configuration of T^4 orientifolds with seven branes. These seven branes are the sources of non trivial τ in this framework (see [15] for more details).

There are many subtleties that we have ignored here, which have been discussed in [15] though. The generic orientifold action in the type IIB case is in fact *ambiguous*. This ambiguity gives different heterotic duals and is present in four- as well as six-dimensional models. Of course, once we fix the heterotic compactification, we also fix the type IIB ambiguity. Nevertheless, the six-dimensional compactification becomes rather involved because of this subtlety. For a more detailed discussion of this issue in the generic context see [27], and for the torsional case, see [15].

This brings us to the next interesting case of a six-dimensional compactification with non zero Euler characteristics. The duality chains from F-theory to the heterotic theory were given in [15]. If we keep the orientifold action Γ as generic, then the type IIB manifold will be of the form \mathcal{N}_6/Γ , where \mathcal{N}_6 is a six-dimensional compact manifold (in the absence of fluxes) that has non zero Euler characteristics. In the presence of fluxes, the heterotic dual of this manifold will look like a Z_2 action of a torus fibration (locally) over a compact base. If we choose another action of Γ that gives a P^1 fibration over a $P^1 \times P^1$ base on the type IIB side (in the absence of fluxes), then the heterotic manifold will be a non-trivial

six manifold that is a Z_2 action of a torus fibration over a $P^1 \times P^1$ base. The generic fibration structure is not difficult to work out (though the analysis may get very tedious). The result, in a compact form, is presented in [15] and therefore we refer the reader to this paper. However, the full topological data have not yet been worked out. This will be presented elsewhere.

The examples discussed above are all of orbifold nature but in principle smooth examples do exist. Some of these have been discussed in [15]. They include the connected sums of $S^3 \times S^3$ (first discussed in [28]) and some examples of flops of an elliptically fibered CY space. The flops used herein are the ones that break the Kähler condition. In [15] it was discussed in detail how the bundles follow the manifold through the flop. But whether these smooth examples are solutions of string equations of motion have not been discussed yet. This is relegated to a future publication.

The question of finding stable vector bundles for our manifolds is a very important one especially because we are no longer allowed to embed the spin connection into the gauge connection (see [29], [11] for a discussion on this). The DUY equations take the same form as in the CY case, i.e the (2,0) and the (0,2) part of the curvature vanishes and the (1,1) part is traceless. However, there are two additional conditions. First, notice that J is no longer closed (and may not be integrable either) and second, there is now a constraint on $\text{tr}(F \wedge F)$ coming from the $i\partial\bar{\partial}J$ part (see [15] for details). These conditions look very restrictive and one might wonder if there exists *any* solution at all to these equations. Again the duality chasing comes to the rescue here. The Weierstrass equation governing the F-theory background allows an D_4^4 bundle to propagate to the heterotic side taking the orientifold action into account. This bundle does satisfy all the conditions as has been explicitly demonstrated in [11]. One has to carefully take the orbifold singularities, localized fluxes *and* higher order corrections in F-theory into account. Failing to do so will not reproduce the correct result [11]. In this way we obtain one particular consistent example, for which the above equations can be solved.

There are several open questions that might be raised at this point. First is the question of the *stability* of the bundle⁸. For Kähler compactifications the holomorphic gauge fields that satisfy the DUY equations are equivalent to Mumford-Takemoto stable holomorphic bundles. For non-Kähler compactifications, assuming the metric to be (approximately) Gauduchon, a similar statement can also be made (though under some

⁸ Earlier work on this appeared in [30].

special circumstances). For more details see [15]. Unfortunately a full understanding of the bundles has not yet been achieved and we hope to address this question in a near future.

Another open question is to understand the full non-abelian nature of the bundle. From the duality chasing one expects the full non-abelian nature to show up. But in practice, it is only the abelian part that is manifest in this scenario (the localized G-fluxes form the Cartan subalgebra of the D_4 algebra). The full non-abelian part can be seen, if we consider the M2 branes to be wrapping the degenerating cycles of Taub-NUT. In [11] the intersection matrices of these two cycles were shown to reproduce exactly the Cartan matrix of the D_4 algebra (at least near one of the four orbifold singularities).

Finally, one might ask about the number of generations for these non-Kähler models. Since we do not allow the standard embedding, the number of generations is *not* equal to the Euler number of the non-Kähler manifold. However we need to evaluate the number of generations when (a) the non-trivial warp factor is taken into consideration, and (b) the spin connection is the torsional connection (in addition to not having standard embedding). Somewhat surprisingly, the number of generations is still given by the third Chern class c_3 of the bundle. For a proof of this see [15]. In [15] the number of generations for a rather simple example with a $U(1)$ bundle has been worked out. For a more realistic non-Kähler compactification one would have to compute the number of generations for a phenomenologically relevant group (say for example $SU(5)$). This would mean that we need an $SU(5)$ bundle on our non-Kähler space satisfying the DUY equations. Whether this is indeed possible remains to be seen. More details on this will be reported elsewhere.

Having a detailed analysis of non-Kähler manifolds, it is now time to ask whether we can formulate a non-linear sigma model description of these manifolds. Some details of this have appeared in the early works of [5],[6], where the basic constraints were shown to follow from a (0,2) sigma model description (for a more recent exposure see [11] and [15]). One would expect a reasonable non-linear sigma model description behind our construction but the calculation of massless spectra is rather subtle here. See [15] for a discussion on this and the issue of \mathcal{H} -twisted sheaf cohomology for the counting of states. However at this point, it is not clear how to formulate a simple *linear* sigma model that flows to the conformally invariant background that we have.

The non-linear sigma model, on the other hand, can be efficiently used to describe various interesting aspects of the torsional backgrounds. In particular one can explicitly derive the preferred connection for this background. The preferred connection is chosen in

such a way that with this choice the heterotic σ -model action resembles the Green-Schwarz superstring action in a background configuration with 24 free fields. The identification becomes precise as soon as we embed the gauge connection in the modified spin-connection. However, as we discussed above, the standard embedding is not allowed here; and therefore only to the lowest order in α' the two actions may be identified. On the other hand, with this choice of connection one can in fact use the Green-Schwarz superstring action to compute $\mathcal{O}(\alpha')$ corrections to the heterotic beta functions. All these corrections are suppressed by the size of the six-manifold, and conformal invariance is restored when the manifold attains the size that is dictated by the minima of our superpotential. For this and other details regarding σ -model description see [6] for an early exposition and [11] for a more recent discussion related to the superpotential. The fact that three form appears on both sides of the Bianchi identity can also be easily shown. The torsional equation and the DUY relations follow from demanding world sheet susy [6],[5], [11]. One can also show that the fundamental form should be \mathcal{H} -covariantly constant, so that the manifold allows an integrable complex structure.

The above discussion, hopefully, summarizes our present understanding of heterotic compactifications on non-Kähler complex manifolds with torsion. We have shown that it is possible to get an almost *rigid* vacua by using background fluxes. To finish this summary let us remark that there is another important issue that has not been addressed so far. This has to do with the *number* of possible string vacua. Recall that at the beginning of this note we discussed two kind of degeneracies: one of them originated from the fact that many different manifolds can be solutions to the string equations of motion and the other one resulted from the deformations of a *given* vacuum. What we discussed so far concerns only the lifting of the second type of degeneracy i.e. the fixing of the moduli for a *given* vacuum. But we have, at present, no understanding on how to fix the other degeneracy. A recent counting of flux vacua in a different context has revealed that this number could be finite in some cases [31]. It would be rather interesting to understand if such a calculation can be performed in the present scenario.

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